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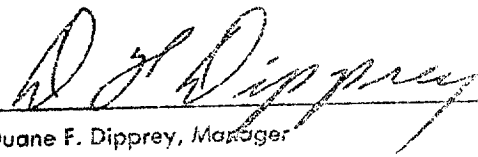
Technical Memorandum 33-395

*Criteria for Separation of Impinging Streams
of Hypergolic Propellants*

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Abstract

A strongly exothermic reaction in the impingement region of two liquid streams can cause so much gas formation that the streams blow apart before substantial mixing can occur. Poor combustion performance in some liquid propellant rocket engines has been traced to this stream separation.

Two theoretical models of stream separation in impinging unlike doublets are advanced. These models are complementary, in that one applies at low pressure and the other at relatively higher pressures. The first model is based on the attainment of the bubble-point temperature at the stagnation point of the jets. In the second model, for higher pressures, the gas phase reactions become so rapid that an insulating gas film can be formed between the two impinging liquid jets, preventing any contact between liquid phases.

Using available data on rates of reaction for the N_2O_4 - N_2H_4 propellant combination, estimates of the region where stream separation would or would not occur have been made. Curves based on the bubble-point limitation depend strongly on the variation of vapor pressure with propellant feed temperature. The gas film model exhibits no such dependence. The $-3/2$ power dependence on pressure of this latter model is due to the sensitivity of gas phase reactions to pressure. The theory is equally applicable to round jet and to flat sheet injector elements. The experimental data obtained at the Jet Propulsion Laboratory appear to correlate with the theory, in that the smaller-diameter injectors (less than 0.02 in.) show no stream separation, but the larger ones (greater than 0.06 in.) do.

Criteria for Separation of Impinging Streams of Hypergolic Propellants

I. Introduction

The occurrence of strongly exothermic chemical reactions near the stagnation point of impinging-stream rocket-engine injectors can materially affect the spray atomization and mixing process in the combustion chamber. Elverum and Staudhammer (Ref. 1) attributed low performance in some types of hypergolic-propellant rocket engines to stream separation. They conjectured that gas evolution due to rapid liquid phase reaction at the impingement point caused the streams to blow apart and separate before a substantial amount of liquid phase mixing could occur. Johnson (Ref. 2) and Evans, Stanford, and Riebling (Ref. 3) have confirmed the effects of stream separation on rocket engine performance. Burrows (Ref. 4) has recently made photographic observations of stream separation in a transparent rocket engine.

In the work reported here, two different but complementary theoretical models of stream separation are postulated. The first model, suggested in Ref. 5, is based on the attainment of the bubble-point temperature at the stagnation points of the jets. In the second model, for higher pressures, the gas phase reactions become so

rapid that an insulating gas film is formed between the two impinging liquid jets. This gas film prevents contact between the liquid phases, thus precluding liquid phase reaction, and also prevents mixing. Using available data on rates of reaction for the N_2O_4 - N_2H_4 propellant combination, predictions are made of the region where stream separation will occur.

II. Liquid Phase Reactions Controlling

Two initially circular cylindrical streams of liquids which impinge are illustrated in Fig. 1. The axes of the two orifices are taken to be coplanar. For steady inviscid flow, a stagnation point will be located on the interface between the two streams. Although the general description of the flow field is complicated, a power series solution for flow in the immediate vicinity of the stagnation point is found to be relatively simple. If one takes the stagnation point as the origin of the coordinate system, the first-order solution for the stream function is given by the relation

$$\psi = \alpha r^2 \bar{y} \quad (1)$$

where y is the length coordinate normal to the interface, r is the radial coordinate measured from the stagnation stream line, and the constant α is a scale parameter whose value is determined by the dimensions of the jets. The exact evaluation of the parameter α would, of course, require the exact solution of the hydrodynamics of the impinging stream. To assess important effects with a minimum of complication, a very simple, completely symmetric geometry for impinging jets has been investigated. This geometry is illustrated in Fig. 2. The analog solution of Leclerc (Ref. 6) for an axially symmetric jet impinging normal to a flat plate is fitted approximately if the scale parameter is chosen to be $\alpha = V_j/D_j$, where V_j is the initial jet velocity and D_j is the initial jet diameter (Ref. 7). In the more general case, the two jets are not identical in size or mass flow, since the propellant ratios are seldom so exactly matched. No study of these effects has been made so far, but some suggestions as to their importance are discussed later.

For liquid phase reaction to occur, there must be mixing between the fuel and the oxidizer streams. It is assumed that this mixing is restricted to a narrow boundary layer and is controlled by turbulence created in the approach flow. This last restriction is believed necessary, because with laminar diffusion controlling the spread of reactants and heat, the heat would be dissipated relatively more rapidly than the mixing reactants could gen-

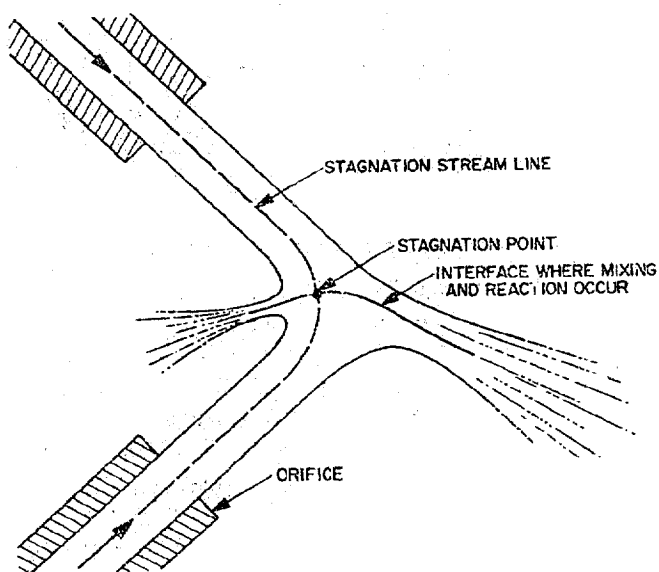


Fig. 1. Diagram of impinging liquid streams showing stagnation stream line, stagnation point, and interface between the two streams

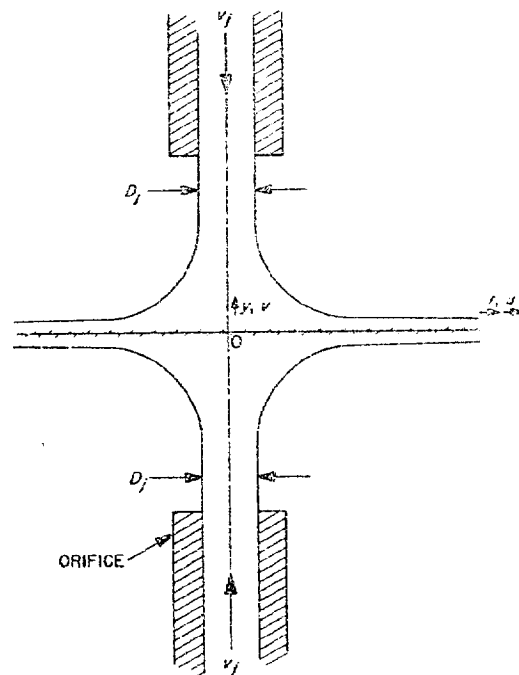


Fig. 2. Diagram of ideal coaxial, opposed impinging streams adopted for a simplified analysis of liquid-reaction-controlled stream separation

erate heat. This effect can be viewed as a consequence of the laminar Lewis number for liquids being much larger than unity. On the other hand, for turbulent mixing, the mechanism of mixing of reactants and the dissipation of heat are the same, and thus the turbulent Lewis number is about unity (Ref. 8). As a consequence, the distribution of reactants and of enthalpy can be assumed to be linearly related and locally adiabatic. Turbulent mixing should cause the calculated maximum temperature level in the reaction zone to be higher than with laminar mixing.

The boundary layer near the stagnation point has a constant thickness (Ref. 9). Consider a "disk-shaped" control volume centered on the stagnation point with the flat sides separated by a distance of the order of the diffusion length. This represents the chemical reactor volume. The residence time for particles passing through this volume varies considerably. Considering the diffusion within a layer of the thickness of the boundary layer, however, it is the average residence time that is the characteristic time duration for the reaction rate and heat release analysis. Using the stream function relation defined in Eq. (1), the average residence time t_r for a particle in this disk-shaped region is given by

$$t_r = \frac{1}{\alpha} \cong D_j/V_j \quad (2)$$

Note that the average residence time t_r does not depend on the dimensions of the arbitrary control volume; this is a property of stagnation point flow. More detailed analysis of the flow field, mass transport, and heat transfer equations can be used to support these contentions.

In lieu of more elaborate analysis, the residence time given in Eq. (2) is assumed as an adequate first approximation to the contact time in impinging jets for chemical reaction.

Directly applicable data on the liquid phase reaction rate of hydrazine and nitrogen tetroxide were not found. As a provisional estimate, some data by Somogyi and Feiler (Ref. 10) on the reaction between hydrazine and nitric acid were used. A reported maximum heat release rate of 83×10^3 cal/s per mole of oxidizer (i.e., N_2O_4) was chosen as an order-of-magnitude estimate.

The stream separation criterion for the case of liquid phase reaction controlling is computed by equating the contact time to the time required to heat a mixture of the reactants to the bubble point. Both streams are assumed to have the same initial temperatures. The total pressure is found by using the law of additive vapor pressure applicable to immiscible fluids. (Hydrazine and nitrogen tetroxide in the liquid forms are believed to be immiscible, Ref. 11.) The time required to heat the mixture from the initial temperature to the bubble-point

temperature is then found by using the previously mentioned heating rate. The total pressure is identified with the chamber pressure, and the heating time is identified with contact time t_r . The results for various initial temperatures are shown in Fig. 3. For an injector with a characteristic contact time $t_r = D_j/V_j$, separated streams may be expected when the chamber pressure is less than that on the curve corresponding to the initial propellant temperatures.

III. Gas Phase Reaction Controlling

If a sufficient amount of heat can be generated by gas phase combustion reaction as compared with the liquid phase reactions previously considered, then there is the possibility that a stable gas film can be formed between the two impinging liquid streams. The gas in the film is generated by vaporization of the liquids due to heat conducted from the combustion zone to the liquid surfaces. The pressure drop resulting from the escape of the gaseous combustion products from the gas film to the environment supplies the force necessary to turn the impinging liquid streams without contact between the liquid phases. The critical consideration in establishing the gas film will be the rate of heat release in the gas phase.

In the case of two coherent liquid jets impinging upon each other, the jets will be assumed, for simplicity, to be exactly opposed with the axes of symmetry coincident (Fig. 4). The momenta of the jets are matched so that the position of the interface will be stationary.

It is assumed that a film of gas separates the two jets at the interface of impingement so that the liquid phases never come in contact. The film of gas separating the two fluids has a thickness δ that is constant, and the gas pressure is P . Fuel is vaporized at one surface and oxidizer is vaporized at the other surface. The mass rate of vaporization is assumed to be uniform over each surface.

The mathematical analysis of uniform blowing from opposed parallel surfaces, including gas phase reaction, can be solved in principle, since the profiles of temperature and concentration are self-similar (Ref. 12). However, the analysis is still quite complex, and hence the temperature and concentration throughout the gas film are assumed to be represented by single mean values. The problem of determining the relation of the mean value to the actual profile will not be considered here.

Sawyer and Glassman (Ref. 13) proposed, on the basis of experimental studies, that the gas phase reaction of

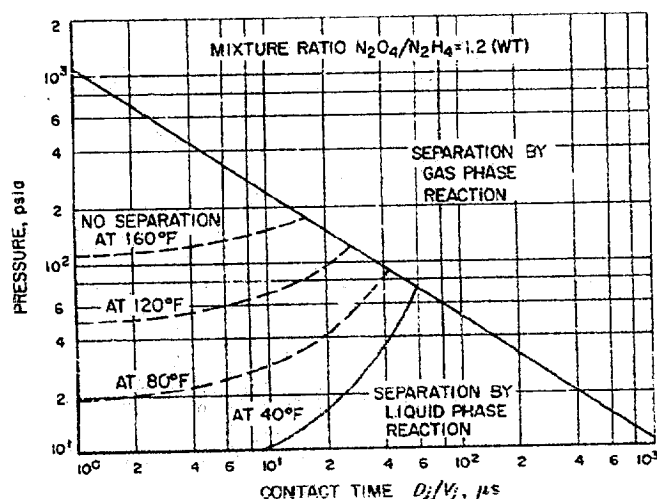


Fig. 3. Stream separation criteria

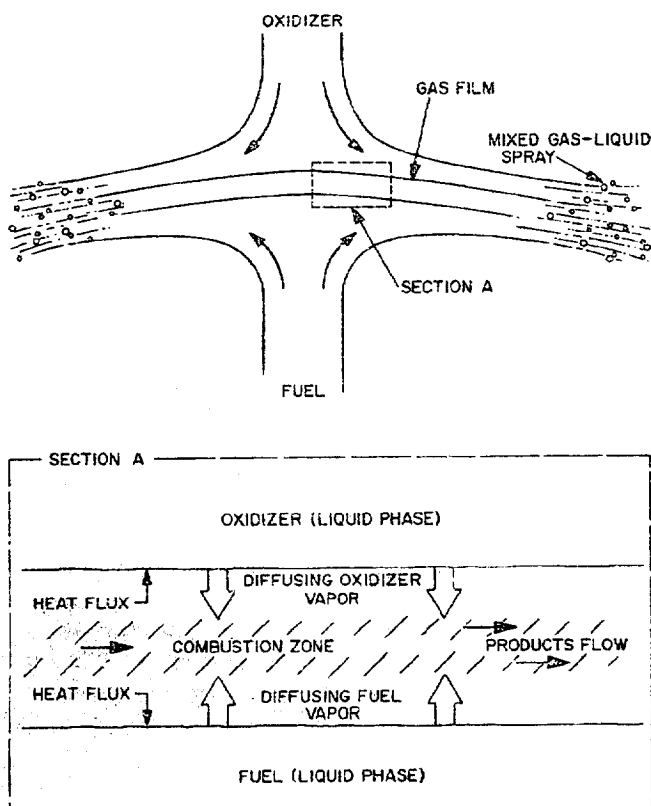
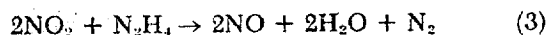


Fig. 4. Diagram of gas film between two reactive liquid streams

hydrazine and nitrogen tetroxide proceeds in two steps. The stoichiometry of the first step, which is very rapid, is



The second step is the slower oxidation of the hydrazine by NO. (It is assumed that only the first step takes place in the gas film.) Sawyer and Glassman found a second-order reaction expression to fit the data:

$$\frac{-d(\text{N}_2\text{H}_4)}{dt} = B (\text{N}_2\text{H}_4) (\text{NO}_2) \quad (4)$$

where the rate coefficient B is given by

$$B = 10^{15.82} \exp(-26,700/RT) \text{ cm}^3/\text{mole-s} \quad (5)$$

In the subsequent analysis, the products of reaction will be taken to be a single species with a mean molecular weight M_m of 24.8.

In terms of mass fraction, the rate expression for the creation of combustion products (subscript m) becomes

$$\frac{dw_m}{dt} = \frac{5B\rho M_m w_f w_o}{M_f M_o} \quad (6)$$

where the number 5 comes from the stoichiometric proportions called out in Eq. (3). The terms M_f , M_o , and M_m are the molecular weight for the fuel, oxidizer, and product, respectively.

To obtain an expression for the rates of gasification, the rate of heat transfer to each surface is equated to the net heat of vaporization. The equations, assuming steady-state conditions, are

$$\bar{\rho} u_f = N \frac{k}{\delta C_p} \left[\frac{C_p (T - T_f)}{L_f + C_f (T_f - T_{f,i})} \right] \quad (7)$$

$$\bar{\rho} u_o = N \frac{k}{\delta C_p} \left[\frac{C_p (T - T_o)}{L_o + C_o (T_o - T_{o,i})} \right] \quad (8)$$

where $N = h\delta/k$ is the Nusselt number, k is the thermal conductivity, T is the gas temperature, T_f and T_o are the surface temperatures of fuel and oxidizer, respectively, $T_{f,i}$ and $T_{o,i}$ are the initial propellant temperatures, and L_f and L_o are the heats of vaporization. The surface temperatures are computed by assuming that the vapor pressure at the surface is equal to the local static pressure (i.e., surface boiling).

Since the only source of gases in the film is vaporization, the equivalent unreacted weight fractions of fuel and oxidizer in the gas phase are given by

$$w_f^* = \frac{\bar{\rho} u_f}{\bar{\rho} u_f + \bar{\rho} u_o} \quad (9)$$

and

$$w_o^* = 1 - w_f^* \quad (10)$$

The actual weight fractions of fuel and oxidizer are given by

$$w_f = w_f^* - \frac{M_f}{5M_m} w_m \quad (11)$$

and

$$w_o = w_o^* - \frac{2M_o}{5M_m} w_m \quad (12)$$

where the subscript m denotes equivalent product for $N_2O_4-N_2H_4$ combustion according to Eq. (3).

An enthalpy balance enables an equation for the mass fraction of products to be written in terms of the gas temperature:

$$w_m = \frac{w_f^*}{L_m} [C_F (T - T_f) + L_f + C_f (T_f - T_{f1})] + \frac{w_o^*}{L_m} [C_F (T - T_o) + L_o + C_o (T_o - T_{o1})] \quad (13)$$

where L_m is the heat of reaction per unit mass of product formed.

The outflow of reaction product from the gas film must match the rate of creation due to reaction; hence

$$(\bar{\rho} \bar{u}_f + \bar{\rho} \bar{u}_o) \pi r^2 w_m = \rho \frac{dw_m}{dt} \cdot \pi r^2 \delta \quad (14)$$

When Eqs. (6)-(8) are combined with Eq. (14), the following expression results:

$$\delta^2 = \frac{N k \Gamma M_f M_o w_m}{5 B \rho^2 C_F w_f w_o M_m} \quad (15)$$

where the parameter Γ is defined as

$$\Gamma = \frac{C_F (T - T_f)}{L_f + C_f (T_f - T_{f1})} + \frac{C_F (T - T_o)}{L_o + C_o (T_o - T_{o1})} \quad (16)$$

The gas density ρ is computed by the perfect gas law

$$\rho = \frac{P \bar{M}}{RT} \quad (17)$$

where the mean molecular weight is given by

$$\bar{M} = \left(\frac{w_f}{M_f} + \frac{w_o}{M_o} + \frac{w_m}{M_m} \right)^{-1} \quad (18)$$

Equation (15) can be solved for the gas temperature T if the film thickness δ and pressure P are stated. However, in view of the complexity of the analysis, it is somewhat simpler to regard δ as the unknown and T as the known parameter.

The gas film thickness δ is determined by the balance of the momentum pressure of the jet compared with the excess of static pressure in the gas film due to friction and acceleration.

The integrated force due to static pressure of the jet along the plane of impingement must match the momentum of the jet. For the purpose of this analysis, it is assumed that the mean static pressure in the gas film is given by the stagnation pressure P_t , which is

$$P_t = P_a + \frac{\rho_j V_j^2}{2} \quad (19)$$

where P_a is the chamber pressure and the subscript j refers to liquid jet properties.

The mass rate of vaporization of gases from the surfaces feeding into the gas film is assumed constant over the surface. By mass balance, the radial velocity of gases in the gas gap at the radius r is given as

$$\bar{\rho} \bar{u} = (\bar{\rho} \bar{u}_f + \bar{\rho} \bar{u}_o) \frac{r}{2\delta} \quad (20)$$

From the partial differential equations for conservation of radial momentum for constant ρ and μ , and from Eq. (20), it can be shown that the laminar pressure drop is given approximately by

$$P_t - P_a = 0.45 \frac{(\bar{\rho} \bar{u}_f + \bar{\rho} \bar{u}_o)^2 r_a^2}{\rho \delta^2} + 3 \frac{\mu (\bar{\rho} \bar{u}_f + \bar{\rho} \bar{u}_o) r_a^2}{\rho \delta^3} \quad (21)$$

The derivation of Eq. (21) is given in the Appendix. Combining Eqs. (7), (8), (15), (16), (19), and (21), and putting $r = D_j$, the expression for the critical time constant D_j/V_j is obtained:

$$\frac{D_j}{V_j} = \frac{0.4 w_m M_f M_o}{\rho B w_f w_o M_m} \left[\frac{\frac{\rho_j}{\rho}}{\left(1 + \frac{20 Pr}{3 N \Gamma} \right)} \right]^{1/2} \quad (22)$$

The transport properties k and μ enter only implicitly into the Prandtl number $Pr = \mu C_F/k$.

It has been assumed that the flow remains laminar, in which case, the Nusselt number N is about 4; if the flow

is turbulent instead, then the Nusselt number is larger than 4. The dimensionless temperature parameter Γ will generally be much larger than unity, because the mean gas temperature will be close to the adiabatic flame temperature. If this is the case, then the value of group $20Pr/3N\Gamma$ will be generally less than unity. Hence, the value of D_j/V_j will be only weakly dependent upon assumptions concerning the nature of the hydrodynamics in the gas film.

Equation (22) is an expression for D_j/V_j , which can be shown to be a function of P and T . Assuming P and T to be given, the mass flows can be computed from Eqs. (7) and (8). The mass flows are used to compute w_f^* , w_o , w_f^* , w_o , and w_m with Eqs. (9)–(13). Finally, D_j/V_j is computed using Eqs. (16), (17), and (18) to evaluate the terms in Eq. (22). The result for the rate expression given in Eq. (5) is plotted in Fig. 5. For each value of D_j/V_j , there can be two, one, or no value of T . There will be a critical value of D_j/V_j , denoted henceforth by $(D_j/V_j)^*$. For values of D_j/V_j larger than the critical, there exist real solutions, while for values smaller than the critical, no real solutions can be obtained. In the case of multiple solutions, it is believed, on the basis of analogy with the behavior of well-stirred adiabatic reactors (Ref. 14), that only the higher-temperature branch represents a physically possible solution. The lower-temperature branch corresponds to a metastable condition.

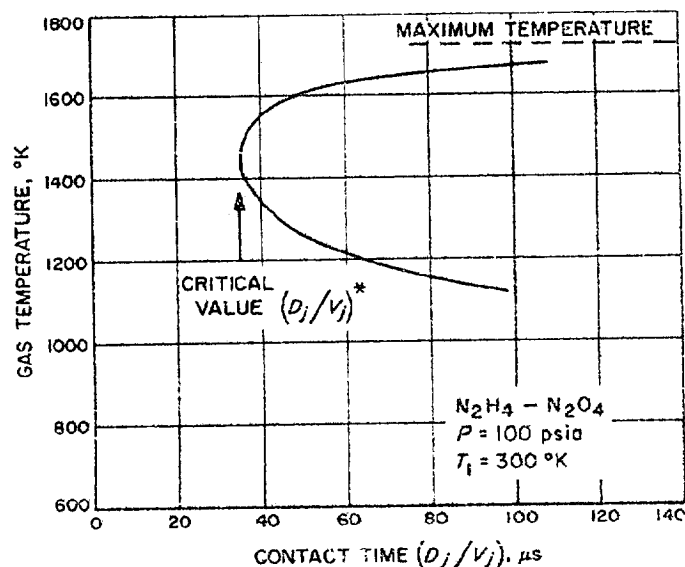


Fig. 5. Contact time as a function of temperature at constant pressure

The existence of a critical value $(D_j/V_j)^*$, below which no real solution exists, is the feature that was sought as the condition for the existence of a gas film between two impinging liquid jets. The critical values $(D_j/V_j)^*$ were computed for different pressures and are presented as part of Fig. 3. The slope of the curve is given by the pressure to the exponent -1.5 .

In summarizing the analytic results, it is proposed that a stable gas film prevents direct liquid phase contact between hydrazine and nitrogen tetroxide liquid jets if the condition

$$\frac{D_j}{V_j} > 35 \left(\frac{100}{P} \right)^{-1.5} \mu s$$

is obtained. The pressure is expressed in psia.

IV. Comparison of Theory With Experiments

As a result of the theoretical analyses, it is possible to outline, in a tentative manner, the regimes in which impinging stream injectors would suffer from stream separation. The data apply to the hydrazine-nitrogen tetroxide propellant system. The criteria are presented as curves (Fig. 3) with chamber pressure and contact time as coordinates. In the case where the liquid phase reaction is controlling, the initial propellant temperature is important, since the temperature rise is directly proportional to the contact time. At a given contact time D_j/V_j , the chamber pressures higher than the vapor pressure would cause a bubble to collapse. The second model, for the gas phase reaction controlling, is essentially independent from whether there can be rapid phase reactions. The separation criterion given in Fig. 3 indicates the lowest pressure at which the gas phase reaction can support a gas film of the postulated type. The combination of the two limits marks out a roughly triangular region of short contact times where no separation effects would be predicted.

The experience at the Jet Propulsion Laboratory on stream separation encountered in rocket engines is described in Ref. 3, which gives a tabular summary for the effect of injector size on separation. These data are compared with the stream separation criteria curves and are presented in Fig. 6. As indicated, the initial propellant temperature was taken to be about 60°F.

Two types of injectors were studied: the tubular orifice impinging jet injectors with an included angle of

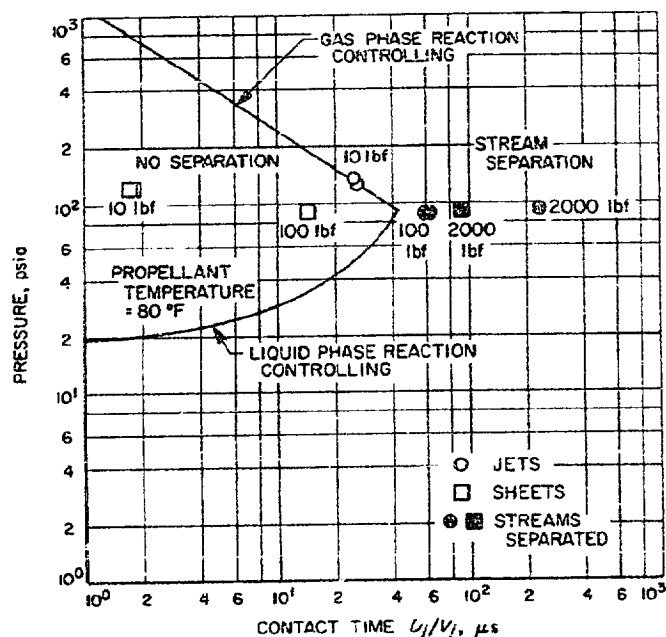


Fig. 6. Experimental data on stream separation

60 deg and the sheet injector, with which the two streams of liquid are deflected to form flat impinging liquid sheets. The characteristic contact times for the sheet injector are computed from the sheet thickness rather than from the diameter. The thicknesses are generally much smaller than the diameter for the comparable tubular injector with the same mass flow per element. The nominal thrust per element is indicated to give some basis for comparison with other engine configurations.

The solid points denote cases where stream separation was found, and the open points indicate those where there was no stream separation. The predicted stream separation criteria do indeed segregate the data into the two types. However, considering the approxima-

tions used in the analysis, the correlation must be considered to be fortuitous.

V. Discussion

The analyses of stream separations presented should be viewed as only a small extension from dimensional analysis. The quantitative predictions have been considerably better than expected. At the present time, it is believed that, until more data are available, the analysis of stream separation should be used as a correlating scheme rather than as a predictive device.

The difficulty in describing the fluid-flow field in the impinging stream injectors was circumvented by restricting the analysis to the stagnation region. The equations then become simple to handle, but an adequate correlation between stagnation point flow and gross jet geometry has been made only for the directly opposed stream. It is hoped that later refinements may clarify the relation between gross stream properties and the effective contact time at the impingement point.

Some estimates have been made of the effect of impingement angle on the contact time (Ref. 5). However, it is not warranted at the present time to include this variable, since most stream half angles are between 30 and 45 deg, a range where the change in values of sines and cosines is less than the probable reliability of the analysis. The effective contact time is not believed to be strongly affected by the impingement angle in this range.

In cases where the impinging stream velocities and characteristic dimensions are not the same as in this analysis, the arithmetic average velocity and the smaller linear dimension should lead to a reasonable estimate of contact time.

Nomenclature

B	rate coefficient	w	weight fraction in gas phase
C_f	specific heat capacity of liquid fuel	y	length coordinate normal to interface
C_o	specific heat capacity of liquid oxidizer	z	vertical distance from centerline
C_P	specific heat at constant pressure	α	scale parameter
D_j	diameter of jet	δ	thickness of gas film
k	thermal conductivity	η	$2z/\delta$
L	latent heat of vaporization	μ	viscosity
M	molecular weight	Γ	parameter
N	Nusselt number	ρ	gas density
P	gas pressure	ψ	stream function
P_a	chamber pressure		
Pr	Prandtl number	Subscripts	
P_t	stagnation pressure	i	initial
r	radial coordinate measured from stagnation stream line	f	fuel
T	temperature	m	equivalent product for $N_2O_4-N_2H_4$ combustion
t_r	average residence time	j	jet
u	radial velocity	o	oxidizer
V_j	initial jet velocity	Superscripts	
v	vertical velocity	$^{\circ}$	equivalent unreacted
		$*$	critical value

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Appendix

Radial Pressure Drop Relation

Equation (21) of the text will be derived herein. To simplify the analysis, the gas temperature is taken to be a constant; hence viscosity is constant, and density is a function of pressure alone. The mass conservation and radial momentum equations are, respectively,

$$\frac{\partial \rho v r}{\partial r} + \frac{\partial \rho u r}{\partial z} = 0 \quad (\text{A-1})$$

and

$$\rho v \frac{\partial v}{\partial r} + \rho u \frac{\partial v}{\partial z} = -\frac{dP}{dr} + \mu \frac{\partial^2 v}{\partial z^2} \quad (\text{A-2})$$

The velocity profile is taken to be laminar:

$$v = \bar{v} (1 - \eta^2) \quad (\text{A-3})$$

where $\eta = 2z/\delta$. The thickness δ and the maximum velocity \bar{v} are constants. Integrating Eqs. (A-1) and (A-2) from $z = -\delta/2$ to $z = \delta/2$, we get from Eq. (A-1) the result

$$\bar{v} = \frac{(\bar{\rho} \bar{u}_o + \bar{\rho} \bar{u}_f) r}{2\rho\delta} \quad (\text{A-4})$$

and from Eq. (A-2) the differential equation

$$\frac{dP}{dr} + \frac{6}{5r} \frac{d}{dr} (\rho r \bar{v}^2) + \frac{12\mu\bar{v}}{\delta^2} = 0 \quad (\text{A-5})$$

Using Eq. (A-4), where $\bar{\rho} \bar{u}_o$ and $\bar{\rho} \bar{u}_f$ are constants, and the perfect gas law, Eq. (A-5) can be integrated from the point $r = 0$, where the pressure is the stagnation pressure P_i , to the point $r = r_a$, where the pressure is equal to the chamber ambient pressure P_a . After considerable manipulation, we can relate the pressure to the radius by

$$\left(\frac{P_a}{P_i}\right)^{2/n} - \left(\frac{P_a}{P_i}\right)^2 = \left(1 + \frac{10}{Re}\right) \frac{0.6 g^2 r^2}{P_i \rho_i \delta^2} \quad (\text{A-6})$$

where we define the quantities

$$g \equiv \bar{\rho} \bar{u}_o + \bar{\rho} \bar{u}_f \quad (\text{A-7})$$

$$Re \equiv \frac{g \delta}{\mu} \quad (\text{A-8})$$

and

$$n \equiv 3 \left(1 + \frac{20}{3 Re}\right) \quad (\text{A-9})$$

Equation (A-6) is expanded in a Taylor series around $P_a = P_i$ and is solved for the pressure drop. We get

$$P_i - P_a = 0.45 \left(1 + \frac{20}{3 Re}\right) \frac{g^2 r_a^2}{\rho_i \delta^2} \quad (\text{A-10})$$

It is assumed that the static pressure in the gas film reaches the chamber pressure P_a at the point where $r_a = D_j$.